1.Uniform Distribution: a. Define the uniform distribution and explain its characteristics. b. A random variable X follows a uniform distribution on the interval [a, b]. Calculate the probability density function (pdf) of X and the cumulative distribution function (cdf) of X

The uniform distribution is a continuous probability distribution where every outcome within a given range is equally likely.

It is characterized by a constant probability density function (pdf) over the entire range of possible values.

Unlike many other distributions, the shape of the uniform distribution does not depend on any parameters other than the range itself.

The uniform distribution is commonly represented by a rectangular shape on a graph, reflecting its constant density.

All intervals of the same length within the distribution range have an equal probability of occurrence.

It is a symmetric distribution, meaning the probability is spread evenly across the range.

The cumulative distribution function (cdf) of a uniform distribution increases linearly over the range, starting from 0 at the lower bound and reaching 1 at the upper bound.

Applications of the uniform distribution include modeling random variables like dice rolls, lottery draws, or selecting random points within a geometric shape.

The uniform distribution is widely used in simulations, statistical modeling, and computer science algorithms.

Despite its simplicity, the uniform distribution plays a fundamental role in probability theory and serves as a building block for more complex distributions.

b. PDF and CDF Calculation:

- Probability Density Function (PDF):
    - If X follows a uniform distribution on the interval [a, b], the pdf f(x) is given by:
      f(*x*)={1*b*−*a*if *a*≤*x*≤*b*0otherwise*f*(*x*)={*b*−*a*10if *a*≤*x*≤*b*otherwise
- Cumulative Distribution Function (CDF):
    - The cumulative distribution function F(x) for X is:
      F(*x*)={0if *x*<*ax*−*ab*−*a*if *a*≤*x*≤*b*1if *x*≥*bF*(*x*)= ⎩⎨⎧0*b*−*ax*−*a*1if *x*<*a*if *a*≤*x*≤*b*if *x*≥*b*

2.Normal Distribution: a. Describe the normal distribution and its properties. b. If a random variable X follows a normal distribution with mean 0μ=50 and standard deviation σ=10, calculate the probability that X lies between 40 and 60.

Description and Properties:

1. The normal distribution, also known as the Gaussian distribution, is a continuous probability distribution characterized by a bell-shaped curve.
2. It is symmetric around its mean, with the mean, median, and mode all coinciding at the center.
3. The normal distribution is fully described by two parameters: the mean (μ) and the standard deviation (σ).
4. It is often used in statistical analyses due to its mathematical tractability and its frequent appearance in natural phenomena.
5. According to the empirical rule (or 68-95-99.7 rule), approximately 68% of the data falls within one standard deviation of the mean, 95% within two standard deviations, and 99.7% within three standard deviations.
6. Normal distributions are commonly observed in nature, such as in the distribution of heights, weights, test scores, and errors in measurements.
7. The normal distribution has tails that extend infinitely in both directions, but the probability of extreme values diminishes rapidly.
8. It is widely used in hypothesis testing, confidence interval estimation, and modeling various phenomena in fields like finance, physics, and social sciences.
9. The central limit theorem ensures that the sum or average of a large number of independent and identically distributed random variables will be approximately normally distributed, regardless of the original distribution.
10. Normality assumptions underlie many statistical techniques, making the normal distribution a fundamental concept in inferential statistics.

Probability Calculation:

- Given X ~ N(μ=50, σ=10), to find P(40 < X < 60):
    - Convert the values to Z-scores using the formula: $Z=X-\mu\sigma Z=\sigma X-\mu$
    - For X = 40: $Z40=40-5010=-1Z40=1040-50=-1$
    - For X = 60: $Z60=60-5010=1Z60=1060-50=1$

- Using a standard normal table or calculator, find the probabilities associated with these Z-scores.
- P(40 < X < 60) = P(-1 < Z < 1), which corresponds to approximately 68% according to the empirical rule.

3. Applications of the Normal Distribution: a. Discuss two real-world applications where the normal distribution is commonly used. b. A company's daily sales follow a normal distribution with a mean of $100,000 and a standard deviation of $20,000. What is the probability that the company's daily sales exceed $120,000?

Applications of the Normal Distribution:

a. **Real-world Applications:**

1. **Financial Modeling:** The normal distribution is extensively used in finance for modeling stock prices, returns, and various financial metrics. It facilitates risk assessment, portfolio optimization, and option pricing.
2. **Quality Control:** In manufacturing and industrial processes, measurements such as product dimensions, defects, or machine outputs often follow a normal distribution. Quality control procedures, such as setting tolerance limits or identifying outliers, rely on the normal distribution assumption.

   Probability Calculation:

- Given mean = $100,000, standard deviation = $20,000, and we want to find P(X > $120,000):
  - Convert $120,000 to a Z-score: $Z=X-\mu\sigma=120,000-100,00020,000=1Z=\sigma X-\mu=20,000120,000-100,000=1$
  - Find the probability associated with Z = 1 using a standard normal table or calculator, which is approximately 0.8413.
  - Therefore, the probability that daily sales exceed $120,000 is approximately 1 - 0.8413 = 0.1587 or 15.87%.

4.Normal Approximation to the Binomial Distribution: a. Explain the concept of normal approximation to the binomial distribution. b. If a binomial distribution has parameters n=100 and p=0.4, approximate the probability that the number of successes is between 35 and 45 using the normal approximation.

Normal Approximation to the Binomial Distribution:

a. **Concept Explanation:**

1. The normal approximation to the binomial distribution is a method used when dealing with large sample sizes in binomial experiments.
2. It relies on the central limit theorem, which states that the sampling distribution of the sample mean will be approximately normally distributed for large sample sizes, regardless of the shape of the original distribution.
3. When certain conditions are met (typically $n * p \geq 5$ and $n * (1 - p) \geq 5$), the binomial distribution can be approximated by a normal distribution.
4. This approximation simplifies calculations and allows for the use of normal distribution properties to estimate probabilities associated with the binomial distribution.

b. **Approximation Calculation:**

- Given n = 100 and p = 0.4, to approximate $P(35 < X < 45)$:
    - Calculate the mean (μ) and standard deviation (σ) of the binomial distribution: $\mu=np=100\times0.4=40\mu=np=100\times0.4=40$ $\sigma=np(1-p)=100\times0.4\times0.6\approx4.899\sigma=np(1-p)=100\times0.4\times0.6\approx4.899$
    - Convert X values to Z-scores using the normal approximation: $Z35=35-404.899\approx-1.022Z35=4.89935-40\approx-1.022$ $Z45=45-404.899\approx1.022Z45=4.89945-40\approx1.022$
    - Use the standard normal table or calculator to find the probabilities associated with these Z-scores.
    - $P(35 < X < 45) \approx P(-1.022 < Z < 1.022)$, which can be calculated from the standard normal distribution.

5.Sampling Distributions and the Central Limit Theorem: a. Define the sampling distribution of a statistic and discuss its importance in inferential statistics. b. State the central limit theorem and explain its significance in statistical inference.

Definition and Importance:

The sampling distribution of a statistic is the probability distribution of that statistic based on all possible samples of a fixed size from a population.

It provides crucial information about the behavior of sample statistics, such as the mean or standard deviation, across repeated sampling from the population.

Sampling distributions play a central role in inferential statistics by enabling the estimation of population parameters, hypothesis testing, and constructing confidence intervals.

Understanding the properties of sampling distributions helps in making accurate statistical inferences about populations based on sample data.

b. Central Limit Theorem and Significance:

The central limit theorem (CLT) states that the sampling distribution of the sample mean (or sum) approaches a normal distribution as the sample size increases, regardless of the shape of the population distribution.

It holds true even if the population distribution is not normal, provided that the sample size is sufficiently large (typically $n \geq 30$).

The CLT is fundamental in statistical inference as it allows for the use of normal distribution properties to make inferences about population parameters, even when the population distribution is unknown or non-normal.

It provides a theoretical basis for many statistical techniques, including hypothesis testing, confidence interval estimation, and regression analysis.

The CLT underscores the importance of large sample sizes in obtaining reliable estimates and making robust statistical conclusions.

It is widely applicable across various fields, including biology, economics, psychology, and engineering, where statistical analyses are common.