

1. **Mean and Variance:** a. Let  $X$  be a random variable representing the number of heads obtained when flipping a fair coin 5 times. Calculate the mean and variance of  $X$ . b. Consider a random variable  $Y$  representing the number of defective items in a batch of 100 produced by a machine, given that the probability of a defective item is 0.05. Calculate the mean and variance of  $Y$ .
2. **Linear Combinations of Random Variables:** a. Suppose  $X$  and  $Y$  are random variables representing the number of successes and failures, respectively, in a series of independent Bernoulli trials with success probability  $p$ . Find the mean and variance of  $Z = X - Y$ . b. Let  $X$  be a random variable representing the number of heads obtained when flipping a fair coin 10 times, and  $Y$  be a random variable representing the number of tails. Find the mean and variance of  $Z = X + Y$ .
3. **Chebyshev's Theorem:** a. A random variable  $X$  has a mean of 50 and a standard deviation of 5. Use Chebyshev's theorem to find an upper bound on the probability that  $X$  lies between 40 and 60. b. Given a random variable  $Y$  with mean 100 and standard deviation 10, use Chebyshev's theorem to estimate the probability that  $Y$  lies within 3 standard deviations of the mean.
4. **Binomial Distribution:** a. A biased coin has a probability of 0.3 of landing heads. If the coin is flipped 10 times, find the probability of getting exactly 4 heads using the binomial distribution. b. In a multiple-choice test with 10 questions, each question has 4 choices, and only one choice is correct. If a student randomly guesses the answers, find the probability that the student gets at least 7 correct answers.
5. **Poisson Distribution:** a. The number of customers arriving at a store follows a Poisson distribution with a mean of 5 customers per hour. Find the probability that exactly 3 customers arrive in the next hour. b. In a city, car accidents occur on average 2 times per day. What is the probability that there are no car accidents on a particular day?