1. **Uniform Distribution:** a. Define the uniform distribution and explain its characteristics. b. A random variable X follows a uniform distribution on the interval [a, b]. Calculate the probability density function (pdf) of X and the cumulative distribution function (cdf) of X.

Uniform Distribution: a. Characteristics of the uniform distribution:

1. The uniform distribution is characterized by a constant probability density function (pdf) over a specified interval.
2. It's symmetric if the interval is symmetric around its mean.
3. The probability of any subinterval within the distribution is proportional to the length of that subinterval.
4. The area under the curve equals 1, ensuring that the total probability is distributed evenly across the interval.
5. The mean of a uniform distribution is the average of the endpoints of the interval.
6. The variance of a uniform distribution is $21212(b-a)2$, where a and b are the endpoints of the interval. b. Probability density function (pdf) and cumulative distribution function (cdf) of a uniform distribution:
7. The pdf of a uniform distribution on the interval [a, b] is given by $=1f(x)=b-a1$ for $a \leq x \leq b$ and 00 otherwise.
8. The cdf of a uniform distribution on the interval [a, b] is $=F(x)=b-ax-a$ for $a \leq x \leq b$, 00 for $x<a$, and 11 for $x>b$.

2.Normal Distribution: a. Describe the normal distribution and its properties. b. If a random variable X follows a normal distribution with mean μ=50 and standard deviation =10σ=10, calculate the probability that X lies between 40 and 60?

Normal Distribution: a. Properties and characteristics of the normal distribution:

1. The normal distribution, also known as the Gaussian distribution, is a symmetric, bell-shaped curve.
2. It's characterized by two parameters: the mean and the standard deviation
3. About 68% of the data falls within one standard deviation of the mean, 95% within two standard deviations, and 99.7% within three standard deviations.

4. It's defined for all real numbers, ranging from negative to positive infinity.
5. The probability density function (pdf) of the normal distribution is given by $=2f(x)=2\pi\sigma 1e-2\sigma 2(x-\mu)2$. b. Calculating probabilities with the normal distribution:
6. To calculate probabilities using the normal distribution, we often use z-scores, which represent the number of standard deviations a value is from the mean.
7. The cumulative distribution function (cdf) of the normal distribution is often tabulated or computed using software.
8. To find the probability that a normal random variable falls within a certain range, we use the cdf.

3.Applications of the Normal Distribution: a. Discuss two real-world applications where the normal distribution is commonly used. b. A company's daily sales follow a normal distribution with a mean of $100,000 and a standard deviation of $20,000. What is the probability that the company's daily sales exceed $120,000?

Applications of the Normal Distribution: a. Real-world applications of the normal distribution:

1. Height of individuals in a population.
2. Measurement errors in scientific experiments.
3. IQ scores in a population.
4. Daily stock returns. b. Using the normal distribution in applications:
5. We can use properties of the normal distribution to make predictions and decisions in various fields.
6. For example, in finance, the normal distribution is used to model stock prices and estimate the probability of certain returns.
7. In quality control, it's used to analyze process variability and set tolerances for product specifications.

4.Normal Approximation to the Binomial Distribution: a. Explain the concept of normal approximation to the binomial distribution. b. If a binomial distribution has parameters n=100 and p=0.4, approximate the probability that the number of successes is between 35 and 45 using the normal approximation

Normal Approximation to the Binomial Distribution: a. Concept of normal approximation to the binomial distribution:

1. The binomial distribution describes the number of successes in a fixed number of independent trials, each with the same probability of success.
2. When the number of trials is large, the binomial distribution approaches a bell-shaped curve that resembles the normal distribution.
3. This approximation is based on the central limit theorem, which states that the sampling distribution of the sample mean approaches a normal distribution as the sample size increases, regardless of the shape of the population distribution.
4. For large n, the binomial distribution can be approximated by a normal distribution with mean $\mu=np$ and variance $\sigma 2=np(1-p)$.
5. The normal approximation is useful when computing probabilities involving the binomial distribution becomes cumbersome or when the exact probabilities are not readily available.
6. It provides a simpler method for calculating probabilities and making inferences about the number of successes in large samples.
7. The normal approximation is more accurate when the number of trials is large and the probability of success is not too close to 0 or 1.
8. It allows statisticians to apply properties of the normal distribution, such as the standard normal table or software, to estimate probabilities for the binomial distribution.
9. The approximation improves as the sample size increases, converging to the exact binomial probabilities as n approaches infinity.
10. However, it's important to note that the normal approximation may not be suitable for small sample sizes or extreme probabilities, where the binomial distribution deviates significantly from the normal distribution.

5.Sampling Distributions and the Central Limit Theorem: a. Define the sampling distribution of a statistic and discuss its importance in inferential statistics. b. State the central limit theorem and explain its significance in statistical inference.

Sampling Distributions and the Central Limit Theorem: a. Importance of sampling distributions:

1. A sampling distribution represents the distribution of a statistic (such as the sample mean or sample proportion) calculated from multiple samples of the same size from a population.

2. It allows us to make inferences about population parameters based on sample statistics.
3. The shape of the sampling distribution depends on the population distribution and the sample size.
4. Sampling distributions are crucial in hypothesis testing, confidence interval estimation, and other statistical analyses. b. Statement and significance of the central limit theorem (CLT):
5. The central limit theorem states that the sampling distribution of the sample mean approaches a normal distribution as the sample size increases, regardless of the shape of the population distribution.
6. This is a fundamental concept in statistics because it allows us to use the properties of the normal distribution to make inferences about population parameters, even when the population distribution is unknown or non-normal.
7. The CLT is essential in hypothesis testing, confidence interval estimation, and other statistical methods.