1. **Sample Space and Events: a. A fair six-sided die is rolled. Define the sample space for this experiment. How many outcomes are there? b. Let A be the event of rolling an even number. List the outcomes in event A.**

Sample Space and Events:

Sample space for rolling a fair six-sided die: {1, 2, 3, 4, 5, 6}.

There are 6 outcomes in the sample space because there are 6 faces on the die.

Event A (rolling an even number): {2, 4, 6}.

Event A contains 3 outcomes, which are 2, 4, and 6.

Each outcome in the sample space is equally likely when the die is fair.

Probability of each outcome in the sample space is 1661.

Probability of event A (rolling an even number) is 36=1263=21.

The probability of an event is a number between 0 and 1, inclusive.

The sum of probabilities of all outcomes in the sample space is always 1.

Event A is a subset of the sample space.

2.Counting Sample Points: a. In how many ways can a committee of 3 people be formed from a group of 8 individuals? b. A password consists of 4 characters, where each character can be a lowercase letter (a-z) or a digit (0-9). How many possible passwords are there?

Using Combinations Formula: Apply the combination formula

$C(n,k)=n!k!(n-k)!$

C(n,k)=

k!(n−k)!

n!

, where

n

n is the total number of individuals and

k

k is the number of individuals to be chosen for the committee.

Substitute Values: Substitute

n=8

n=8 (total individuals) and

k=3

k=3 (committee size) into the formula.

Calculate Factorials: Compute the factorials involved:

8!=8×7×6

8!=8×7×6 and

3!=3×2×1

3!=3×2×1.

Apply Combination Formula: Plug the calculated factorials into the combination formula:

C(8,3)=8!3!(8−3)!

C(8,3)=

3!(8−3)!

8!.

Simplify Factorials: Simplify the factorials:

C(8,3)=8×7×63×2×1

C(8,3)=

3×2×1

8×7×6

Perform Calculations: Perform the arithmetic:

C(8,3)=3366

C(8,3)6

336.

Calculate Result:

C(8,3)=56

3. Probability of an Event: a. A bag contains 6 red balls, 4 blue balls, and 5 green balls. If a ball is randomly selected, what is the probability of selecting a blue ball? b. An experiment has 18 equally likely outcomes, of which 5 are favorable to event A. What is the probability of event A?

The probability of selecting a blue ball: 46+4+5=4156+4+54=154.

There are 6 red balls, 4 blue balls, and 5 green balls in the bag.

The total number of balls in the bag is 6 + 4 + 5 = 15.

The probability of selecting a blue ball is the ratio of the number of blue balls to the total number of balls.

Probability of event A: 518185.

There are 18 equally likely outcomes.

Event A has 5 favorable outcomes.

The probability of an event is the ratio of the number of favorable outcomes to the total number of outcomes.

The probability of an event is always between 0 and 1, inclusive.

If an event cannot occur, its probability is 0. If an event is certain to occur, its probability is 1.

4. Additive Rules, Conditional Probability, and Independence: a. Two cards are drawn without replacement from a standard deck of 52 cards. What is the probability that the second card drawn is a king, given that the first card drawn was a queen? b. Are the events of rolling a 4 on a fair six-sided die and flipping a head on a fair coin independent? Justify your answer.

Additive Rules, Conditional Probability, and Independence:

The probability of drawing a king as the second card, given that the first card drawn was a queen: 351=117513=171.

There are 3 kings remaining in the deck after drawing a queen.

There are 51 cards remaining in the deck after drawing the first card.

Events are independent if (∩)=()×()$P(A \cap B)=P(A)\times P(B)$.

The probability of rolling a 4 on a fair six-sided die is 1661.

The probability of flipping a head on a fair coin is 1221.

The events of rolling a 4 on a die and flipping a head on a coin are not independent because 16×1261×21 is not equal to the probability of both events occurring together, which is 0.

5. Random Variables and Probability Distributions: a. Define a random variable and provide an example of a discrete random variable. b. Consider a continuous random variable X that represents the height of a randomly selected person from a population. Describe the probability distribution of X.

Random Variables and Probability Distributions:

A random variable is a function that assigns a real number to each outcome of a random experiment.

It's denoted by capital letters, like X or Y.

It can be discrete or continuous.

Example of a discrete random variable: Number of heads obtained when flipping a coin 3 times.

The possible values of a discrete random variable form a countable set.

The probability distribution of a discrete random variable is described by a probability mass function (pmf).

A probability mass function gives the probability of each possible value of the random variable.

Probability distribution of a continuous random variable (X representing the height of a person):

It's described by a probability density function (pdf).The total area under the pdf curve is 1.

It's represented by functions like the Gaussian or normal distribution.

It's characterized by parameters such as mean (μ) and standard deviation (σ).

It's symmetric around the mean.

It's unbounded, extending from negative to positive infinity.

The probability of obtaining a precise value for X is technically zero.

The probability of X being in a particular interval is given by the integral of the pdf over that interval.